Math CAMPPP 2011

Plenary Three

Multiple Representations

Fostering Student Conjectures

Linking Important Math, Expectations, Learning Goals and Tasks

What Are “Multiple Representations?”

- We don’t mean: numbers, pictures and words
- We are talking about how representations illustrate, deepen, and connect student understanding
  - What does the linear growing pattern representation illustrate about “steepness” for example? (relationship between tile building and graphing)

Connecting Linear Growing Patterns and Graphs

- CLIPS animation (Notebook File)

Connecting Linear Growing Patterns and Graphs

- How does an understanding of linear growing patterns help you to make the connection between the multiplier of a pattern rule and the trend line on the graph?

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Multiple Representations that Further Student Thinking

- Grounding an understanding of algebraic relationships in concrete, abstract and graphical representations allows students to construct symbolic expressions
- For instance, an understanding of both linear growing patterns and graphs can support students in solving equations of the form

  $ax+b=cx+d$

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Sequence of Tasks

- The following tasks were designed to allow students to explore relationships *among* pattern rules through working with both concrete materials and graphs

Task 1

- Predict what the graphical representation for these two pattern rules would look like:

number of tiles = position number **x5+3**

number of tiles = position number **x6+2**

At your table, build the two patterns with tiles, and then graph your results on the chart paper. Don't forget to PREDICT first. And JUSTIFY your prediction. THEN start building!

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Key Questions For You

- How did the concrete tiles representations connect to the graphical representations?
- What conclusions did you draw from this activity?

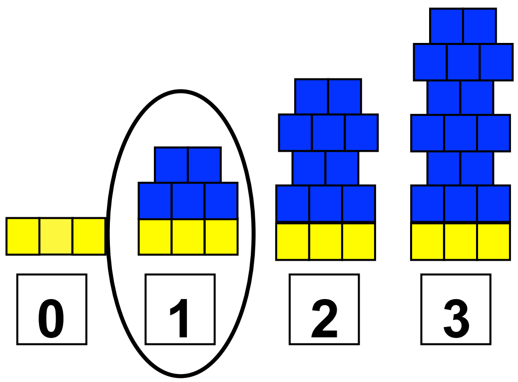
  (specific to the task)

  (more general)

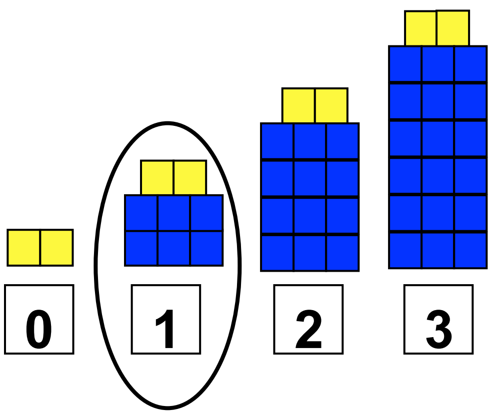
What Did Students Think About?

Ilse: At the first position they're going to be the same because…um… that rule adds up to 8 and that rule adds up to 8. They're going to be the same value.

Amy: It's like if you were building two patterns, based on the two rules, they would both have the same amount of tiles at that position. So that's what it looks like when you graph it, both the lines would have a dot at 8 tiles for position 1.



number of tiles = position number **x5+3**

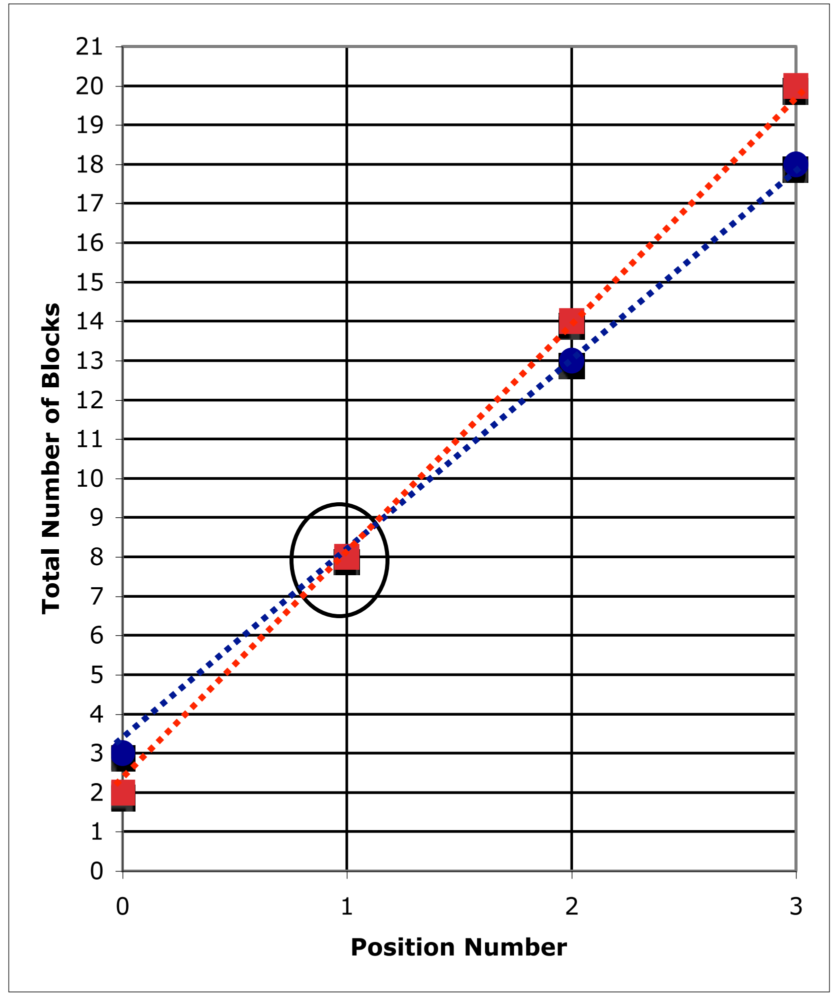


number of tiles = position number **x6+2**

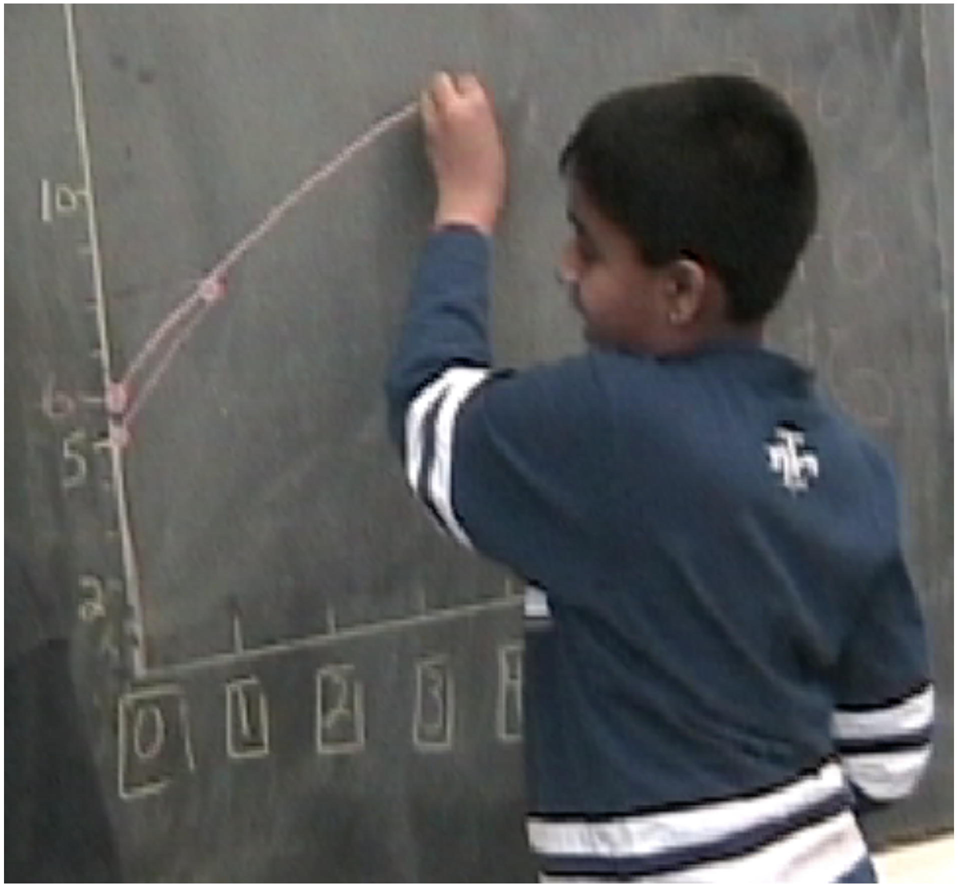
What Did Students Think About?

Anne: The x5+3 is going to start higher because the plus 3 is a higher number so it would start higher but not grow as fast. The x6+2 would be steeper because x6 is a higher value, and so would mean a steeper line on the graph because it's growing faster.

Jack: So they will intersect like *pow, smack, boom*! They're going to intersect at 1 and then keep going!



Drawing On Prior Learning



This line [3x+5] and this line [2x+6] are going to intersect for sure because “times 3 plus 5” for [x-value] one is going to be 8 and for “times 2 plus 6” it’s also going to be 8.

So, the zeroth position is going to be 5 for this one [drawing a point at 5 on the *y*-axis] and 6 for this one [drawing a point at 6 on the *y*-axis] and then it’s going to go something like this [sketches the two trend lines].

But I know it’s going to intersect here [pointing at (1, 8)] for sure because for position 1, both answers are 8!

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Two Student Conjectures

1. If two rules have values that add to the same amount, they will intersect at an x-value of 1
2. To intersect, one rule has to have a higher multiplier and a lower constant, and the other rule has to have a lower multiplier and a higher constant

Testing the Second Conjecture

- What if one rule has a higher multiplier AND a higher constant than the other rule?
- Will the trend lines intersect?
- For example:

number of tiles = position number **x1+2**

number of tiles = position number **x3+4**

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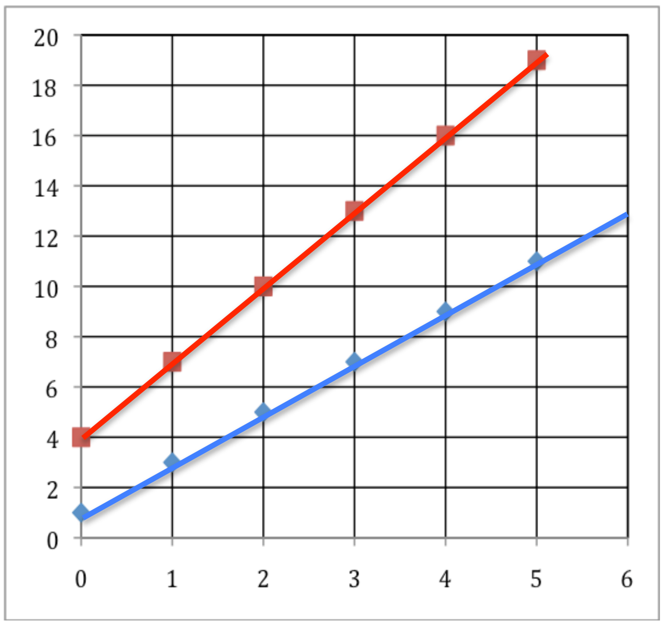
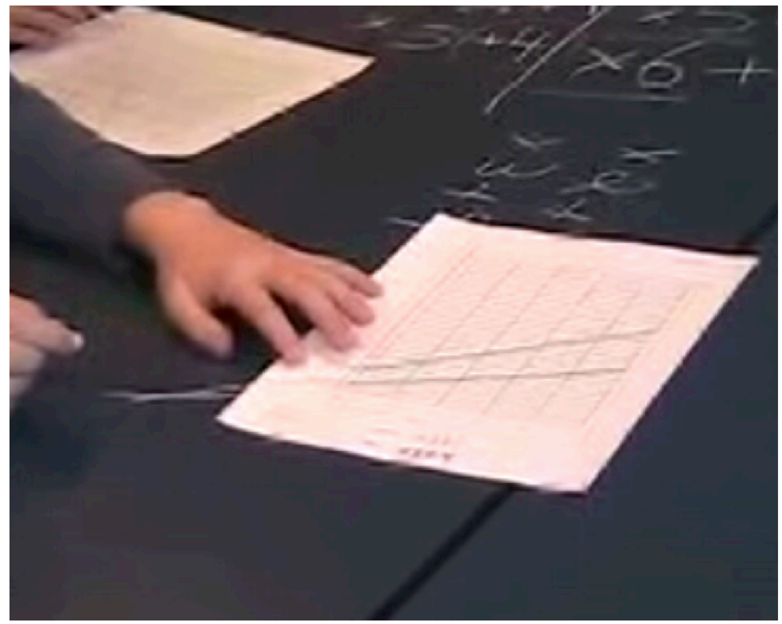
Testing the Second Conjecture

- They don’t intersect…but…they’re not parallel…





Revising the Second Conjecture

- If one rule has a higher multiplier AND a higher constant, they will intersect "somewhere behind zero…like an x-value of negative 2!"
- If two rules have different values for the multiplier and the constant, they will intersect *somewhere*

Linking Important Math, Expectations, and the Task

Important Math (Enduring Understanding)	Expectations	TASK	Learning Goal	Evidence of Understanding
Multiple representations of pattern rules help us understand the relationships between different rules, and find the point of intersection. E.g., two rules that have a constant and multiplier that sum to the same number will intersect when x =1	Grade 9 • apply data-management techniques to investigate relationships between two variables; • demonstrate an understanding of the characteristics of a linear relation; • connect various representations of a linear relation Grade 10 model and solve problems involving the intersection of two straight lines	Use multiple representations to investigate the relationship between two rules such as: x5 + 3 x6 + 2	Explore how the intersection of two trend lines can be represented and identified using multiple representations	1. Students can use their understanding of patterns and graphs to predict “what would happen” 2. Students articulate the meaning of the point of intersection 3. Students make conjectures based on their findings that move beyond the task

Task 2

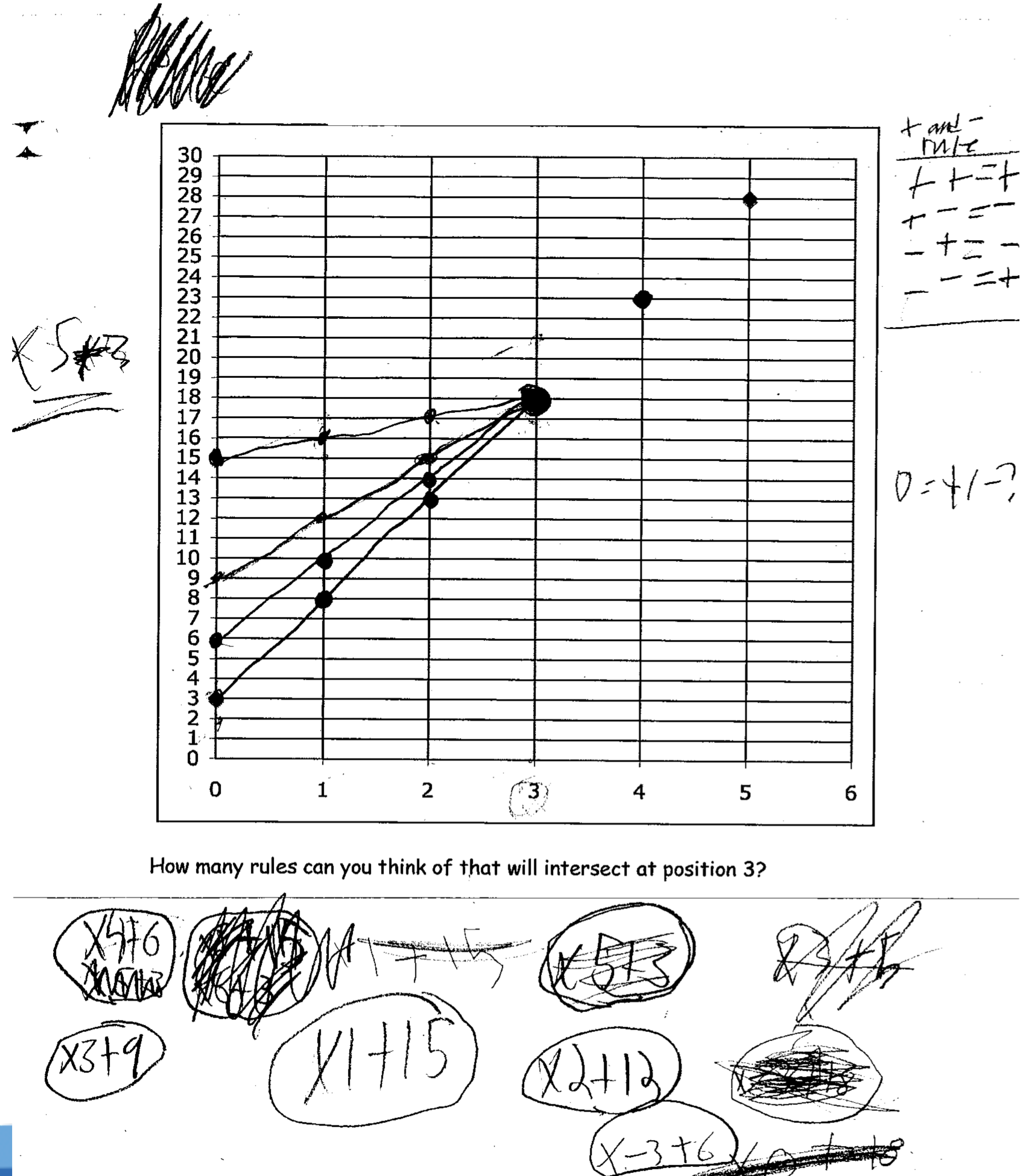
- Given a graphical representation of

  number of tiles = position number x5+3

  how many rules can you think of that would have a trend line that intersects at (3,18)?
- Work with your table group in pairs or in threes

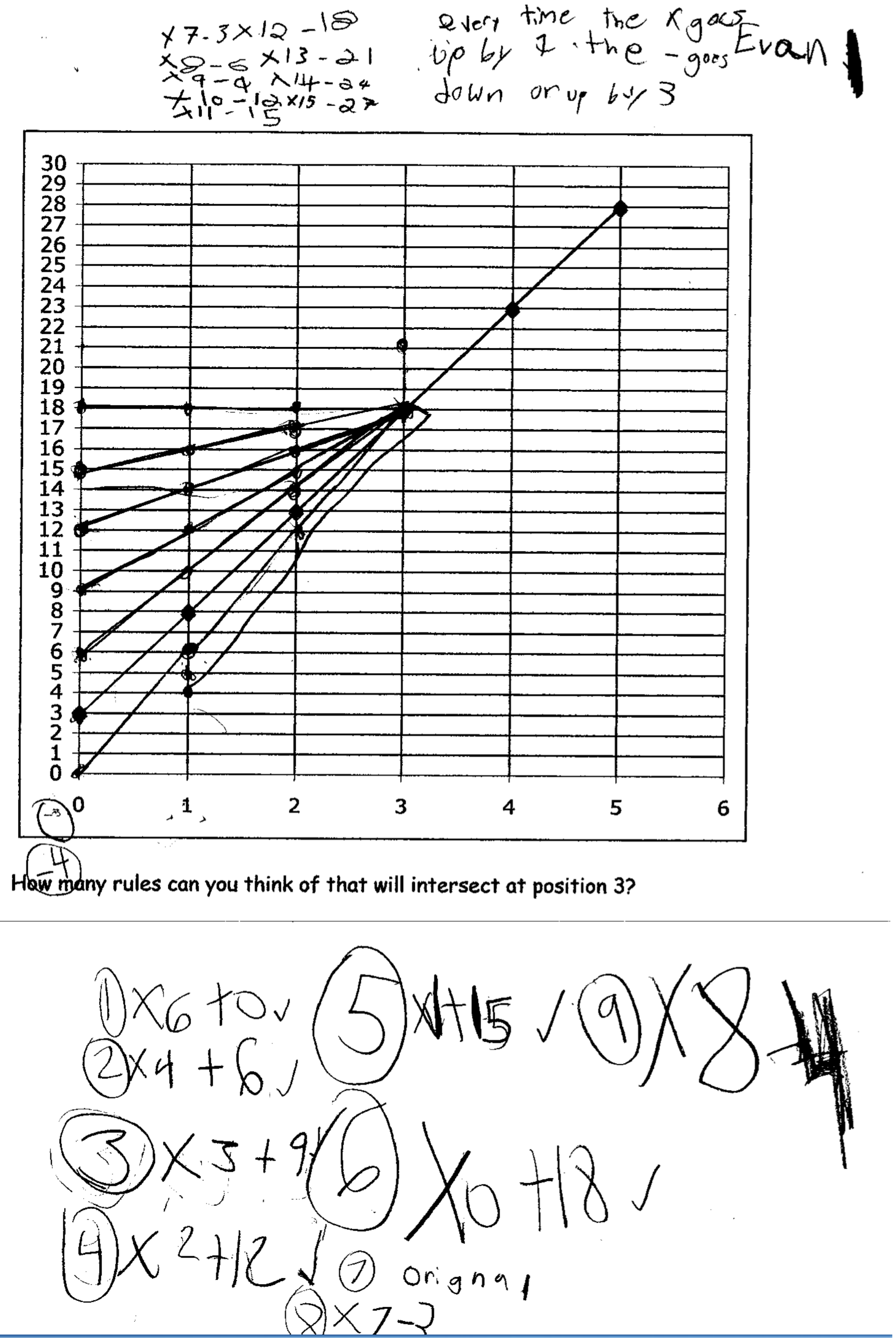
Guess and Check Strategy

- These students thought of different rules and then graphed them to check which ones “worked”
- Rules that didn’t work were crossed out



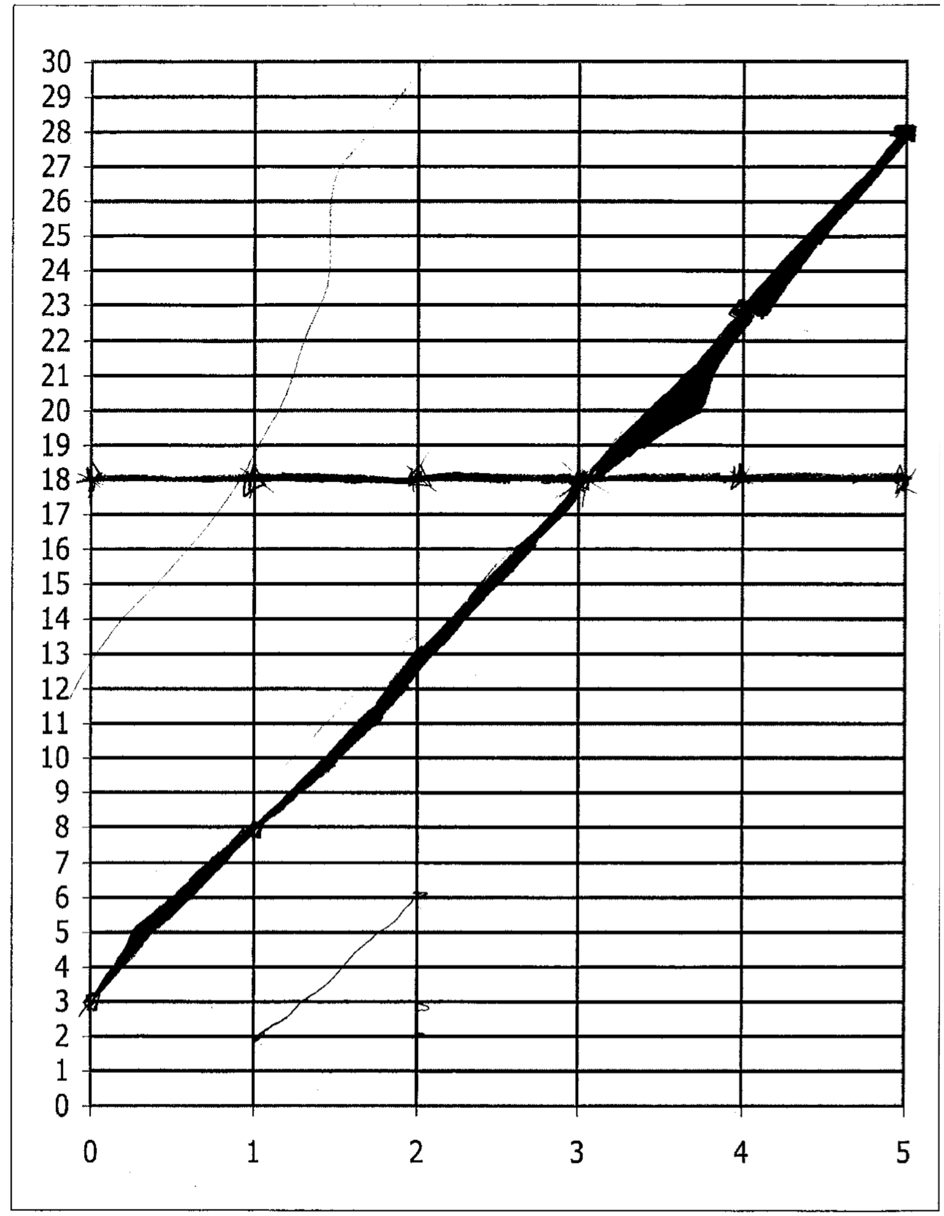
Incorporating Negative Numbers

- Simultaneously guessing and checking rules by drawing the trend lines
- This student started to include rules that had a negative constant



Formulating Equations

- This student, after plotting the trend lines for one rule [x0+18] realized that all the rules would have to have multipliers of 3 (because the point is at x-value 3) and all the rules would have to equal 18 (the y-value)
- She created three equations with a box indicating an unknown constant



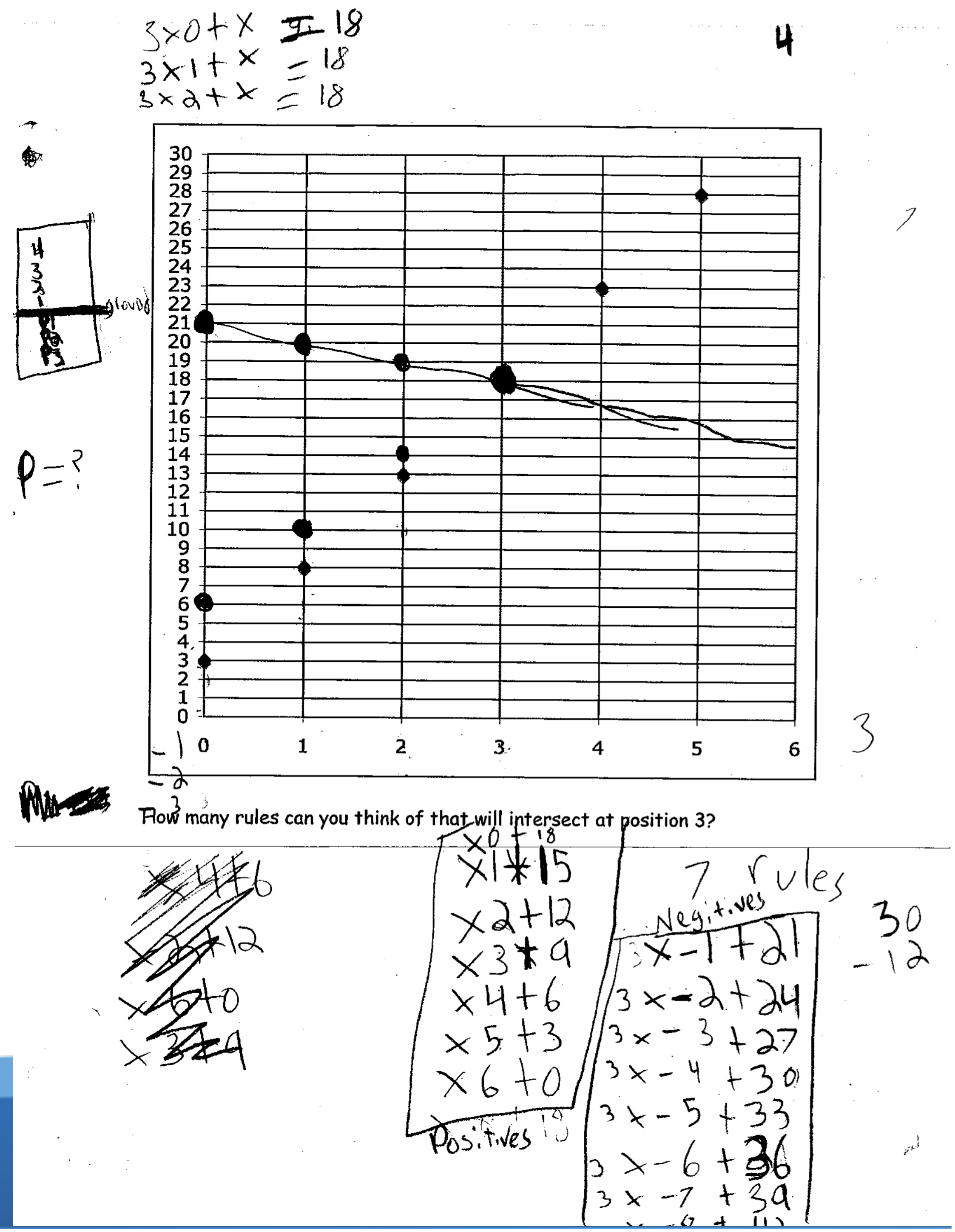
$3 \times 1 + \square = 18$

$3 \times 2 + \square = 18$

$3 \times 3 + \square = 18$

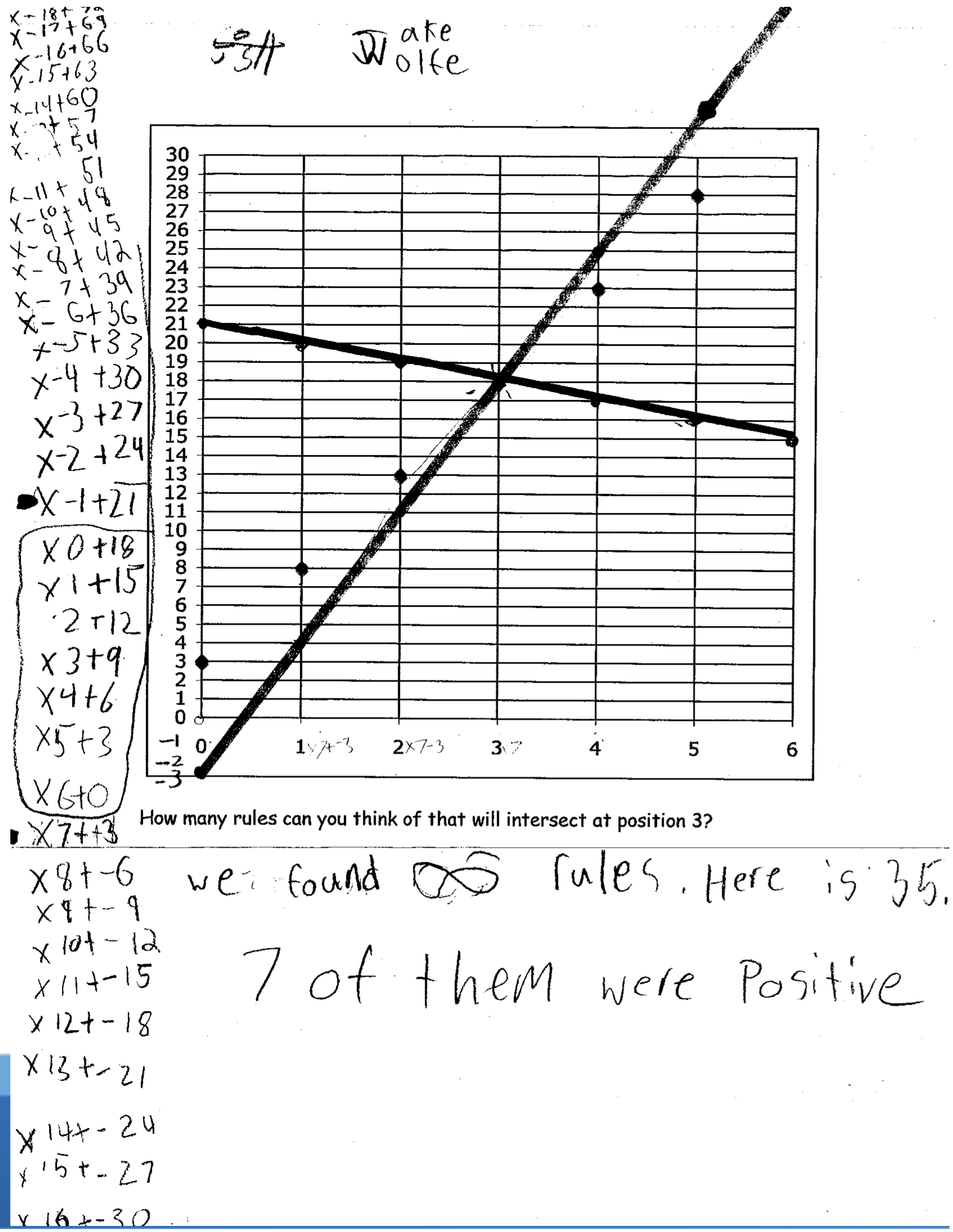
Ordered Equations

- This student noticed a numeric pattern: as the value of the multiplier increased by 1, the value of the constant decreased by 3
- She extended this pattern to list rules with a negative multiplier (one of which she has graphed, since it was a new concept for her)



Exhaustive List

- These students also figured out the numeric pattern and then explored rules that had a negative multiplier (one of which they graphed) or a negative constant (one of which they graphed)



Student Learning Supported by Task 2

- Students had a deeper understanding of the relationship between pattern rules and trend lines
- Most students created equations with two unknown variables: 3 x ___ + ____ = 18
- Some students began playing with negative values for the multiplier and the constant, and then explored what this could look like on a graph

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If…

- If we look at rules that have whole numbers, what pattern do you see (let’s include 0)?

$0x+18$
$1x+15$
$2x+12$
$3x+9$
$4x+6$
$5x+3$
$6x+0$

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Supporting Conjectures

- What needs to be true of two rules if their trend lines are to intersect in the first quadrant?
- What part of the rule determines where the line starts (or is at on the y-axis?)
- What part of the rule determines the slope (or "how steep" the line is)?

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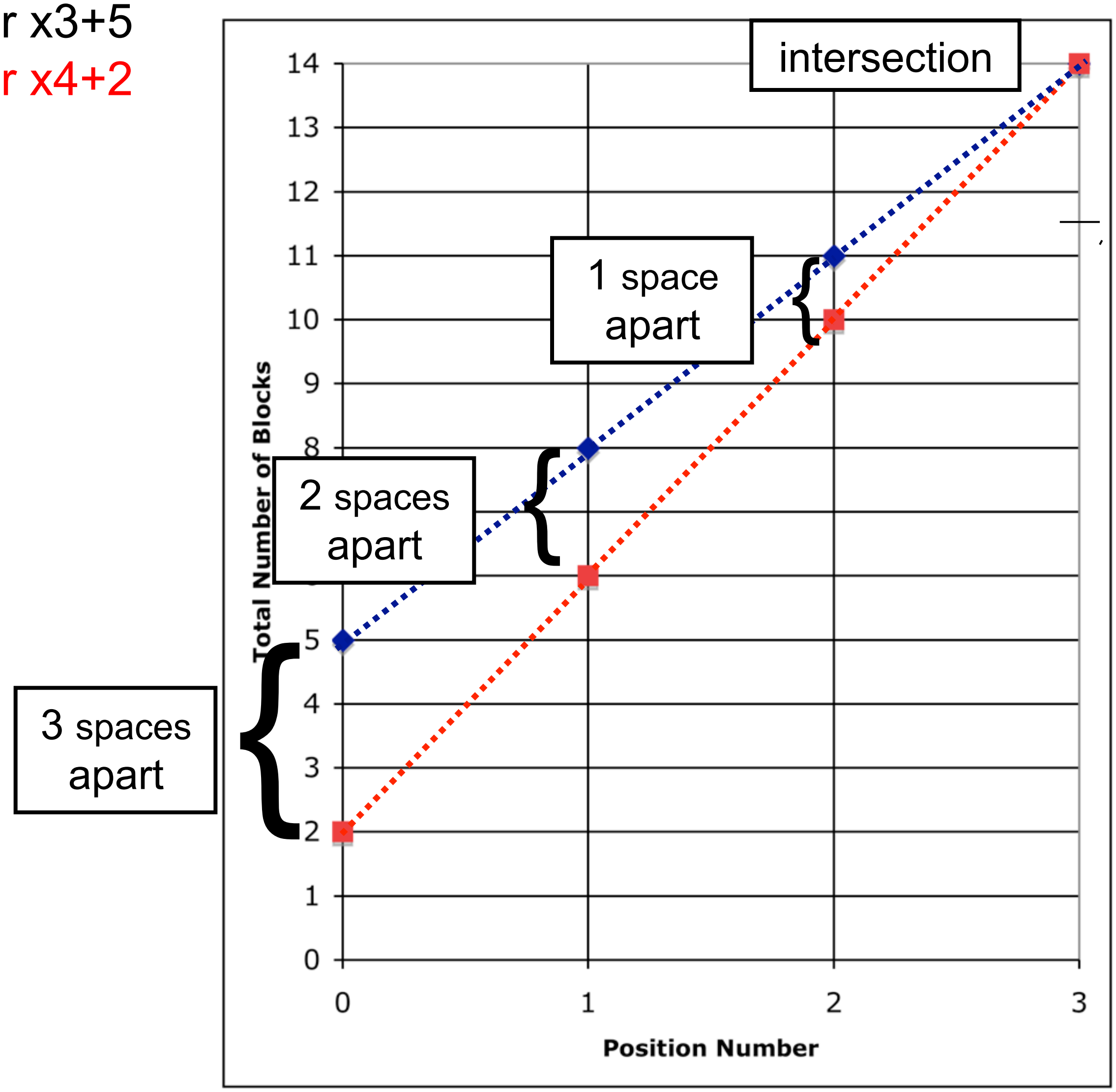
New Student Conjecture

3. If the multipliers of two rules differ by one, then the difference between the value of the constants is same as the x-value of the point of intersection

number of tiles = position number x3+5

number of tiles = position number x4+2

Pete: For each one (trend line) you have to think about how far apart they're starting on the graph and how long it will take them come together. So if the rules are x3+5 and x4+2, they start three spaces apart and get together by one space each time, so it would take them to the third position to intersect.



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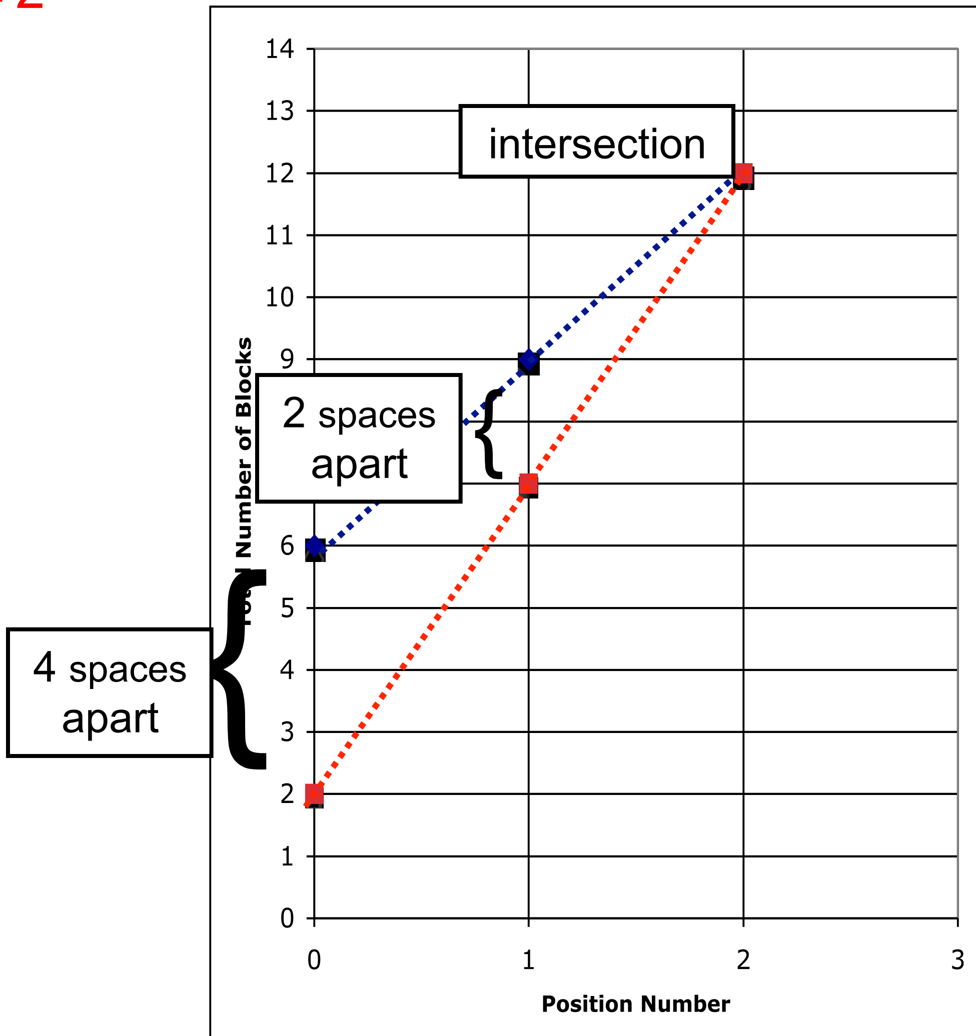
number of tiles = position number x3+6

number of tiles = position number x5+2

Alan: So then say you have multipliers that are two apart. The trend lines would come together by 2 spaces each time. So if they start 4 spaces apart, and then come together by 2 each time, they'll intersect at position 2.



Final Conjecture

“For any two rules that will have trend lines that intersect in the first quadrant, if you know how far apart they ‘start’ by comparing the value of the constants, and the rate at which they come together by comparing the value of the multipliers, you can predict where the trend lines will intersect - or the point at which the two rules will have the same position value (x) and tiles value (y).”

Solving Equations

- If you think about it, solving an equation like 3x+5=x+16 is like thinking about the position number (x) where two patterns would have the same number of tiles (y)
- It's also like thinking about the x-value where two trend lines would intersect on a graph, if you knew how far apart they "started" and how quickly they were coming together

“Balancing” Equations

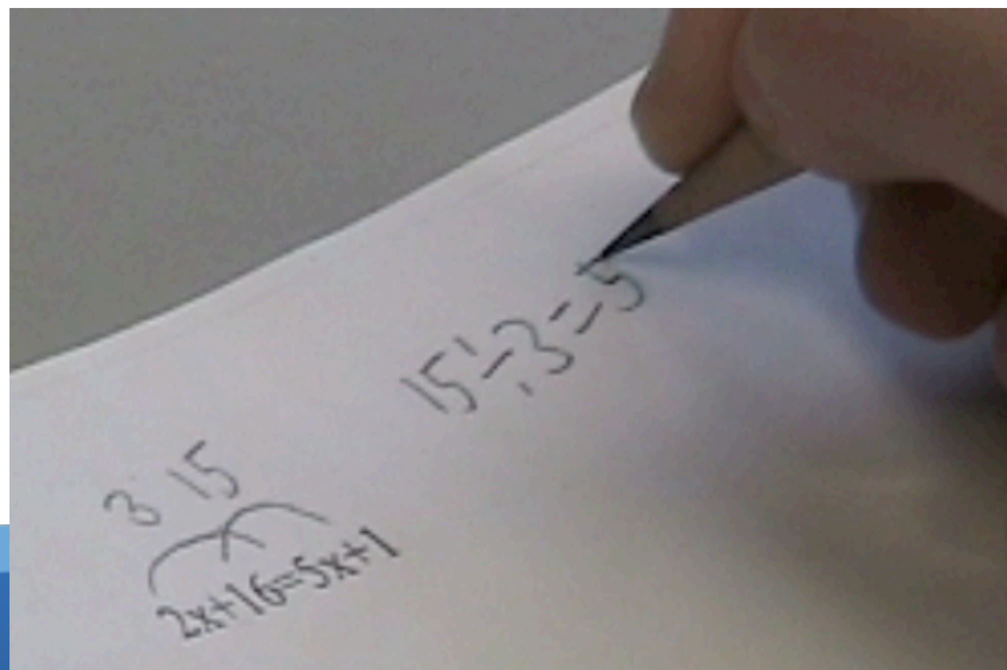
- An explanation of how to solve equations of the form ax+b=cx+d

difference of **3**

$$2x+16 = 5x+1$$

difference of **15**

$$15 \div 3 = 5$$



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Solving Equations

- In this video, John is able to work only with equations, with numbers greater than those he worked with previously
- Given his understanding, the values of the equations can be larger than might be possible when working with pattern tiles
- He can check his calculations by using the graph – but in this example he does not need to create the entire graph – he uses only one point on the graph to indicate the point of intersection

Supporting Student Thinking

- Allow students to explore the connections among different representations – pattern rules, patterns, graphs, and equations
- Include both active and passive graphing activities

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Supporting Student Thinking

- Promote the offering and evaluation of conjectures
  - Is it always the case that this is true?
  - Can you think of a counter-example?
  - If we introduce a new idea, how does that affect the conjectures we already have? [For instance, students had a conjecture that the constant was responsible for "where the trend line starts." When they started using a 4-quadrant graph, this was amended to "the constant is responsible for the y-intercept"]

Responding To Student Thinking

- Ask good questions, and help them to ask their own good questions
  - What part of the rule is responsible for the slope?
  - What part of the rule is responsible for the y-intercept?
  - How can you determine whether two rules will have trend lines that intersect?
  - How can you determine the point of intersection for the trend lines of any two rules?